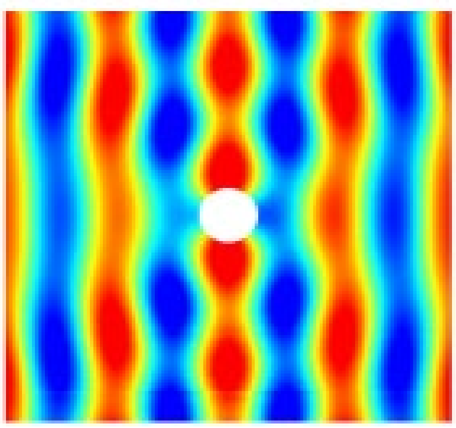
M&MIMS24 International Workshop

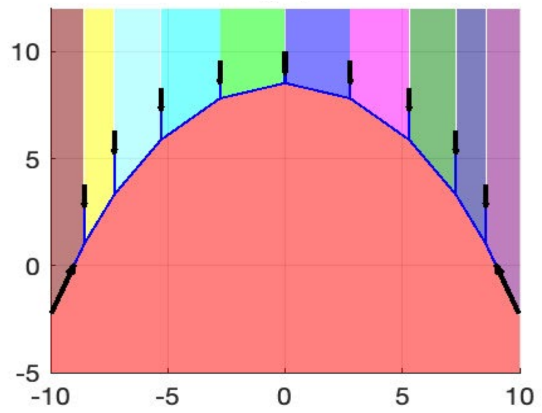
“Mathematics and Mechanics of Innovative Materials and Structures”

Multimedia Room, Strength Laboratory, University of Salerno

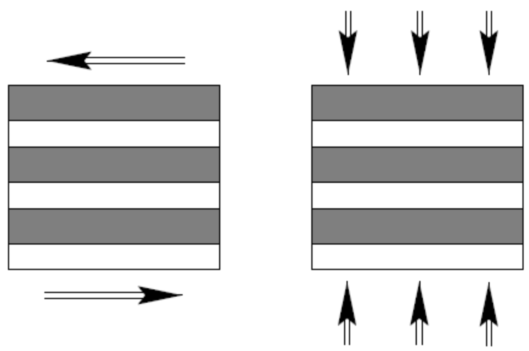
Keynote Lectures by Professor Graeme Milton
Department of Mathematics, University of Utah (USA)

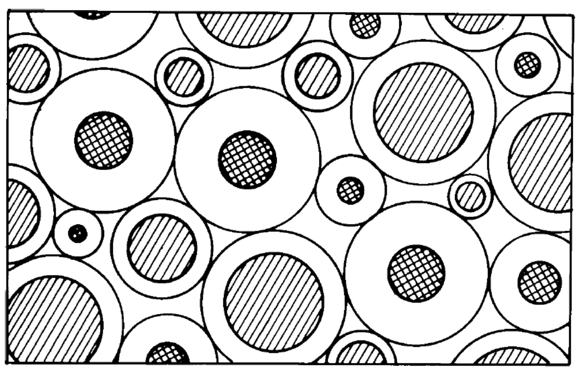
Lecture 1 : July 25, 2024 – h 15:00-16-00:
Guiding Stress : From Pentamodes to Cable Webs to Masonry Structures





Lecture 2: July 25, 2024 – h 16:15-17.15
What elastic responses can composites have?





Lecture abstracts

Guiding Stress : From Pentamodes to Cable Webs to Masonry Structures

This lecture addresses the following key question pertaining the elastic behavior of composite materials: for Pentamode materials are a class of materials that are useful for guiding stress. In particular, they have been proposed for acoustic cloaking by guiding stress around objects, and have been physically constructed. A key feature of pentamode materials is that each vertex in the material is the junction of 4 double cone elements. Thus the tension in one element determines the tension in the other elements, and by extension uniquely determines the stress in the entire metamaterial. Here we show how this key feature can be extended to discrete wire networks, supporting forces at the terminal nodes and which may have internal nodes where no forces are applied. In usual wire or cable networks, such as in a bridge or bicycle wheel, one distributes the forces by adjusting the tension in the wires. Here our discrete networks provide an alternative way of distributing the forces through the geometry of the network. In particular the network can be chosen so it is uniloadable, i.e. supports only one set of forces at the terminal nodes. Such uniloadable networks provide the natural generalization of pentamode materials to discrete networks. We extend such a problem to compression-only 'strut nets' subjected to fixed and variable nodal loads. These systems provide discrete element models of masonry bodies, which lie inside the polygon/polyhedron with vertices at the points of application of the given forces ('underlying masonry structures'). In particular, we solve the two-dimensional problem where one wants the strut net to avoid a given set of obstacles, and also allow some of the forces to be reactive ones. This is joint work with Ada Amendola, Guy Bouchitté, Andrej Cherkaev, Antonio Fortunato, Fernando Fraternali, Ornella Mattei, and Pierre Seppecher.

What elastic responses can composites have?

The elasticity tensor is a fourth order positive definite tensor satisfying required symmetries that can be represented in three dimensions as a 6 by 6 symmetric positive definite matrix. We posed the question as to whether any such matrix can be realized in some elastic material. The answer, which turned out to be "yes, there is" required the introduction of a special class of materials, that we called extremal elastic materials. These have 6 by 6 matrices having eigenvalues falling into two groups with the ratio between eigenvalues in the first group and those in the second group being extremely large. Examples include isotropic materials with the ratio of bulk modulus to shear modulus being very small (an auxetic material having Poisson's ratio close to -1) or very large (a pentamode material). Here we discuss limitations of this result. This then leads to the question of what effective elastic tensors can an isotropic material with voids have when the moduli of the isotropic material are known? Here we discuss progress on this question, and also limitations of the answers, as well as on the affiliated question as to the possible values the average stress can take when the average strain is prescribed. This is joint work with Marc Briane, Mohammed Camar-Eddine, Andrej Cherkaev, and Davit Harutyunyan.

Short bio

Graeme W. Milton received B.Sc. and M.Sc degrees in Physics from the University of Sydney (Australia) in 1980 and 1982 respectively. His research at Sydney University focussed on electromagnetic wave propagation through ceramic metal composites, with applications to Solar Energy. He received a Ph.D degree in Physics from Cornell University in 1985 for research in statistical physics, on models exhibiting anomalous phase transitions, and for work in composite subsequently went to the Caltech Physics Department as a Weingart Fellow, from 1984 to 1986, continuing on to the Courant Institute of Mathematical Sciences as Assistant and Associate Professor of Mathematics. He is currently Professor of Mathematics at the University of Utah, Salt Lake City, Utah. His current research includes analysing the effective properties of composite materials, with particular interest in investigating new mathematical techniques which generate sharp bounds on the effective parameters, and finding the microgeometries which attain the bounds. He has studied questions relating to the electromagnetic and elastic properties of composites, and to the properties of fluid-filled porous rocks. He has held both a Sloan Fellowship and a Packard Fellowship.